

**Research to Practice Lesson Plan**  
**Reciprocal Teaching in Math with Paper Slides**  
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**Introduction**

As a math teacher in an alternative high school for four years, my students were often lower in math than their peers at other schools. They had not been successful in math under a “traditional model” of notes, practice, homework, so I was challenged to find ways to engage the students in the content. A complicating feature of this problem was the wide range of skill within one classroom. Grade level equivalency scores on a school-wide assessment for a 10th grade Geometry in the 2014-2015 school year ranged from grade 3- grade 12. Thus, differentiation of instruction became a primary concern.

One tactic I employed frequently was what I had been calling “peer teaching”, where I would take four (generally well-performing) students to whom I would teach the material for the day while the rest of the class was doing the warm up. These four became the “teachers” for the day. I divided the rest of the class into four groups and assigned each a teacher. All groups worked on the same general skill but each group had a slightly different problem type. For example, when teaching graphing, one group might have equations of the form  $y = mx + b$ , another’s problems would all have  $b = 0$  such that their equations take the form  $y = 8x$  or  $y = 3x$ ; a third group’s equations would all be in the form  $x = 5$  or  $x = -6$ , etc. I then set a timer for 10 minutes and the “teachers” teach their assigned skill to their group using whiteboards and dry erase markers. When the buzzer rings, the groups rotate and the timer is reset. I am then available to answer specific questions and monitor behavior. At the end of the 83 minute class period, students have generally done 40 or so problems and been exposed to at least four different explanations of how to do the problem.

When the group work is done, I would assign a small problem set to assess how well the skill was acquired during the process. I have felt that this method effective, refreshing and rewarding for both myself and my students. It adds variety and the students who are teachers are forced to practice patience (or not), learn the material better, explain their thinking to other students and the process brings to the surface any aspects of the content that they do not fully understand. This “peer teaching” method was also helpful in planning because I taught four different subject each day with no repeated sections. Peer teaching offered me a way to reduce time spent planning.

In the next school year I will be teaching at a traditional junior high where most students are at or near grade level. I will have 4 sections of the same class, which presents opportunities for students to review the work of their peers in other classes. This lesson is an attempt to maximize this opportunity and synthesize variety, technology and elements of reciprocal teaching into a cohesive activity that will be fun, engaging and valuable to the students.

### **Investigation & Pedagogy**

I decided to investigate the validity of the interventions I was already using, specifically peer teaching, to see if my hunch that it is an effective strategy has any empirical research behind it. This led me to a paper from 1984 by Palincsar & Brown on a reading intervention called reciprocal teaching. An example of reciprocal teaching in action can be viewed [here](#).

Reciprocal teaching is a protocol in which a teacher models for a student or a small group of students 4 specific interventions: summarizing, questioning, clarifying, predicting. The explanation from the original authors follows.

The basic procedure was that an adult teacher, working individually with a seventh-grade poor reader, assigned a segment of the passage to be read and either indicated that it was her turn to be the teacher or assigned the student to teach that segment. The adult teacher and the student then read the assigned segment silently. After reading the text, the teacher (student or adult) for that segment asked a question that a teacher or test might ask on the segment, summarized the content, discussed and clarified any difficulties, and finally made a prediction about future content. All of these activities were embedded in as natural a dialogue as possible, with the teacher and student giving feedback to each other.

Initially, the adult teacher modeled the activities, but the students had great difficulty assuming the role of dialogue leader when their turn came. The adult teacher was sometimes forced to construct paraphrases and questions for the students to mimic. In this initial phase, the adult teacher was modeling effective comprehension-monitoring strategies, but the students were relatively passive observers. (p. 124-125)

I believe that reciprocal teaching can transfer to a mathematics classroom, though modification may be required. Ultimately I would like to try to replicate the setting outlined above from the original study by Palincsar & Brown in 1982 and 1984, but by the time I discovered reciprocal teaching, I would have needed to rework my entire lesson. Prudence seemed wise and I opted instead to incorporate the four interventions of summarizing, clarifying, questioning and predicting into my current lesson but change the order to clarifying (what is this problem asking me to do? Do

I know what to do first? Is there anything I don't understand here?), predicting (What do I think I will get for an answer? What are the overall steps I need to do?), questioning (Why do I need to do that? Asking someone else, Why did you do that?), and summarizing (Here is what it asked me to do, this is what I did, and this is why).

### *Summary as an Intervention in Math*

My school district is a heavy proponent of the AVID program, which emphasizes summarizing. I did not find studies that were specifically concerning the effect of summarizing mathematics work on student achievement. However, I am keeping it in my lesson because 1) I believe that since summarizing has been proven an effective reading comprehension strategy, that it can transfer across disciplines to mathematics; 2) There may be studies out there I have not discovered; and 3) AVID is supported by my school and we are strongly encouraged to use as much of the program as possible.

### *Paper Slides*

This is an activity that allows students to explain their thinking in a fun, creative way. It also provides them with tangible evidence of learning which can contribute to a portfolio that can be timeless. An example of paper slides can be seen [here](#).

According to the 3rd Standard of Mathematical Practice, mathematically proficient students "justify their conclusions, communicate them to others, and respond to the arguments of others." Paper slides provides a medium through which students can explain their reasoning and fulfill this standard.

Furthermore, Kramarski & Mevarech (2003) say that "Research in cognitive psychology has shown that learning occurs when the learner is engaged in some sort of cognitive restructuring or elaboration (Wittrock, 1986). One of the most effective means of elaboration is explaining the material to someone else." I believe that the vehicle of creating and watching other students paper slides will engage students in this cognitive restructuring.

### **Objective**

The goals of this lesson are for students to:

- Understand that there are different ways to solve a system of equations
- Be able to ask questions that clarify summarizing, clarifying, questioning and predicting in a mathematical context
- Create something with a meaningful audience in mind
- Articulate their thinking on a math problem

- Give and receive feedback to and from their peers
- Have a fun, engaging experience with math class

### **Standards Addressed**

#### Common Core State Standards (CCSS) for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.
- Attend to precision.

#### Common Core State Standards Grade 8 Math Content Standards

- CCSS.MATH.CONTENT.8.EE.C.7.A  
Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers).
- CCSS.MATH.CONTENT.8.EE.C.8  
Analyze and solve pairs of simultaneous linear equations.
- CCSS.MATH.CONTENT.8.EE.C.8.A  
Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- CCSS.MATH.CONTENT.8.EE.C.8.B  
Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
- CCSS.MATH.CONTENT.8.EE.C.8.C  
Solve real-world and mathematical problems leading to two linear equations in two variables.

### **Content**

I have chosen to teach understanding and solving systems of equations for this lesson for three primary reasons.

First, solving systems of equations is a skill that has both a well-defined procedural component as well as an extensive contextual component which is covered in the CCSS. I wanted to have both of these pieces because I think the procedural

component will be easier to teach and a good starting point. I can teach this to everyone to get a baseline of knowledge for each class. The more extensive application problems allow for differentiation of instruction. Additionally, I theorize that since these problems generally require reading comprehension, they will provide students with a bridge that may make the transfer of reciprocal teaching from reading to math easier.

My second reason for choosing systems of equations is that, among the material covered in the 8th grade CCSS is the concept of three types of possible answers: one answer, no solution and an infinite number of solutions (CCSS.MATH.CONTENT.8.EE.C.7.A). These are three general, clear ideas that can easily be divided among two groups of students. I suggest teaching one set of students problems requiring them to solve systems of equations using the substitution method that have one solution and the other set of students problems requiring them to solve systems of equations using the elimination method that have one solution and an infinite number of solutions.

Finally, this content lends itself to the format I wanted to use, which is using video as a means by which students can teach each other.

### **Time Required**

This lesson is intended to take 4 45-minute class periods, but can be amended to be longer. A secondary idea I had was to have it stretch over multiple weeks, devoting one class period a week for the project.

### **Procedure**

#### Prior to Day 1 of Project

1. Divide students into two groups, preferably in different classes or in a capacity where they are taught separately. Further divide each class into pairs. I will pair students with mixed mathematical abilities together, as this seems to benefit the lower achieving students without detriment to the higher achiever. Linchevski, Liora, and Bilha Kutscher (1998) found that "the achievements of our average and less able students proved to be significantly higher when compared to their peers in the same-ability classes, whereas highly able students performed about the same." (p. 533)
2. [Pretest](#) both groups of students (called A & B). Mark these papers correct/incorrect but no more than this. The pretest will be the same assessment as the posttest and will therefore serve as a measurement of

instructional effectiveness. Students will also use their pretest to determine their content problems for this assignment.

## DAY 1

*Warm up:* Give students 2-3 problems regarding linear equations, including graphing, slope and transforming them.

*Classwork:* Give an overview of the project, announce the pairings, and pass out the problems for each pair. Hand the [pretests](#) back so that students can see their own baseline. Hartley and Davies (1976) in a review of literature on preinstructional strategies suggest that pretests benefit students the most when used to inform students of the gaps in their knowledge.

Use direct instruction to teach each section their respective skill of solving systems of equations by elimination or by substitution. I debated a more social-constructivist model where students teach themselves using whatever tools they could find (i.e. the internet, videos, textbook, each other) but decided in favor of direct instruction after reading that in a meta-analysis of discovery learning practices that Alfieri, Brooks, Aldrich & Tenenbaum (2011) “suggest that unassisted discovery does not benefit learners, whereas feedback, worked examples, scaffolding, and elicited explanations do.” (p. 1)

Students will do a 6-8 long problem set as practice to be completed in class.

*Closure:* Have students summarize what they have learned in class today.

## DAY 2

*Warm up:* Give all students in one class the same 2 problems on their learned topic and go over answers.

*Classwork:* In their pairs, students will practice working out and explaining their thinking to each other on various problems using the four interventions in reciprocal teaching in the revised order of *clarifying, predicting, questioning, summarizing*.

Since a key component of reciprocal teaching is the modeling of the interventions is transfer of cognitive responsibility to the student (p. 123), I will model with a student for the class how this might look. This is a larger group than the original study, but I do not have the capabilities at this time to have other adults in the room with me.

Students will then be given a [handout](#) with sample questions to ask their peers as they go through the the process of explaining their reasoning to their partners.

Hand out problems to partners and assist groups as they practice.

*Closure:* Students summarize what they have done and learned in class today.

### DAY 3

*Warm Up:* 1 problem of learned skill.

*Classwork:* Students are assigned a problem for their paper slides. They will then use this class period to create the paper slides using their mobile phone or available school devices. Upload videos to private classroom youtube channel.

*Closure:* Students summarize what they have done and learned in class today.

### DAY 4

*Warm Up:* 1-2 problems of learned skill.

*Classwork:* In the same partners, watch video or screencast from the other class. Students are to watch the video through once without writing. On the second viewing they should work through the problem with the video, working with their partners to clarify and ask questions with. We will rotate through as many problems as time allows.

*Closure:* Students are given 1-2 problems to solve using their new given skill.

### DAY 5

*Warm Up:* 1-2 problems of newly learned skill.

*Classwork:* Class is given time to ask questions to clarify as a group. Give students in both classes the same assessment used for the [pretest](#). This will NOT be a test grade, but rather a check for understanding.

*Closure:* Student feedback of the entire process. Here I am looking for feedback on how well they worked with their partner, if they liked this, what problems (logistical, mathematical, social) that came up; and if this was a meaningful learning experience for them.

## **Discussion**

I want my students to come away with a deep understanding of how to solve a system of equations and what it means for a system to have one, no or an infinite number of solutions. I further want them to struggle with explaining their thinking, learning from each other, and creating something with technology.

This was the first lesson in which I have tried to justify all of my pedagogical decisions with empirical research, and as I got deeper into the lesson the more I became aware of the difficulty of this task. Teaching even one day is a series of decisions made with the best information available at the time. At the end of this lesson, I am left with decisions that remain backed up with my experience and anecdotal evidence from others, but without clear evidence of student benefit.

Some of the questions that remain or that came up in the process are

- What part does creativity play in learning?
- How can I better implement reciprocal teaching with only one adult in the room?
- Should I have students make a handout detailing questions for the four interventions of clarifying, predicting, questioning and summarizing?
- How could I better lay the groundwork for this activity using the student's prior knowledge?

## Resources

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